

Observers and Locality in Everett Quantum Field Theory*

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Abstract

A model for measurement in collapse-free nonrelativistic fermionic quantum field theory is presented. In addition to local propagation and effectively-local interactions, the model incorporates explicit representations of localized observers, thus extending an earlier model of entanglement generation in Everett quantum field theory [M. A. Rubin, *Found. Phys.* **32**, 1495-1523 (2002)]. Transformations of the field operators from the Heisenberg picture to the Deutsch-Hayden picture, involving fictitious auxiliary fields, establish the locality of the model. The model is applied to manifestly-local calculations of the results of measurements, using a type of sudden approximation and in the limit of massive systems in narrow-wavepacket states. Detection of the presence of a spin-1/2 system in a given spin state by a freely-moving two-state observer illustrates the features of the model and the nonperturbative computational methodology. With the help of perturbation theory the model is applied to a calculation of the quintessential “nonlocal” quantum phenomenon, spin correlations in the Einstein-Podolsky-Rosen-Bohm experiment.

Key words: Everett interpretation, quantum field theory, locality, Deutsch-Hayden picture, Einstein-Podolsky-Rosen-Bohm experiment

*This work was sponsored by the Air Force under Air Force Contract FA8721-05-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the author and are not necessarily endorsed by the U.S. Government.

1 Introduction

1.1 Bell's theorem, the Everett interpretation, and locality

It is one of the virtues of the Everett or “many-worlds” interpretation [1] of quantum theory that Bell's theorem [2] does not apply to it. An implicit assumption of Bell's theorem is that a measurement has a unique outcome. In the Everett interpretation more than one outcome can occur. So, “in the framework of the MWI [many-worlds interpretation], Bell's argument cannot get off the ground [3].” Everett quantum theory is therefore not demonstrated by Bell's theorem to be nonlocal [3]-[20].

This still leaves open the question of whether Everett quantum theory in fact possesses the locality property that Bell's theorem denies to single-outcome quantum theory. That property can be summarized as follows:

A theory is local if it can explain correlations in the outcomes of spatially-separated measurements as being due to information carried by some physical process in a continuous fashion through space, at a finite speed, from the location of a common cause [21] to the locations of the measurements in question.

Consider Bohm's version [22] of the Einstein-Podolsky-Rosen [23] experiment (EPRB). Alice measures the spin of a spin-1/2 particle emitted in the decay of a two-particle system in the singlet state. Bob measures the spin of the other spin-1/2 particle. Each obtains one of two possible results, “spin up” or “spin down.” They repeat their experiment many times and with various relative orientations of their respective spin analyzers. For each run of the experiment they start with a new pair of particles in the singlet state, and each time they record their respective results as well as the respective orientations of the spin analyzers used in that run. After many repetitions Alice and Bob compute the correlations between their results. Bell theorem states that, for general choices of relative orientations of the analyzers, there is no explanation for the correlations which Alice and Bob obtain which is local in the sense defined above.

Everett quantum theory avoids Bell's theorem by denying that correlation exist between the measurement results *per se*. After a run of the experiment has been performed, there are no longer two experimenters but four, “Alice-who-saw-up,” “Alice-who-saw-down,” “Bob-who-saw-up” and “Bob-who-saw-down.” Contrary to the situation in single-outcome quantum theory, there is no well-defined notion of the correlation between Alice's and Bob's results until they convey the information about what they have measured to some common location.¹ E.g., Alice and Bob must take their results to Corry, who compares them and, after many runs of the experiment, computes the correlations.

¹Of course the correlation between measurement results in single-outcome quantum theory cannot be *known* without such communication.

Can the correlations that Corry records be explained locally, i.e., in terms of information carried, during each run of the experiment, from the site of preparation of the singlet-state pair and the sites of the measurements (or from any other location containing information about the orientations of the spin analyzers used in the measurements)? To answer this question in the affirmative, the theory must contain some feature corresponding to the idea of “carried by some physical process in a continuous fashion through space etc.,” e.g. local partial differential equations. But, in addition, the theory must also contain local elements in its mathematical formalism—i.e., elements in some way connected at each time to some location in space—corresponding to the physical entities which carry the information. To be able to speak of something being “carried through space,” there must be a “something,” at some place in space, to be “carried.”

1.2 The Deutsch-Hayden picture and quantum field theory

Deutsch and Hayden [24] have shown that the information-carrying elements in question are the time-dependent operators in the Heisenberg-picture version of quantum theory. The information is encoded in the operators by transformations corresponding to the respective interactions which, e.g., in the EPRB case above, entangle the two particles into the singlet state and measure their respective spins at the locations of Alice’s and Bob’s analyzers. In order to make the argument that all of the relevant information is contained in such operators, Deutsch and Hayden introduce a variant of the Heisenberg picture. The Deutsch-Hayden picture is obtained from the usual Heisenberg picture by a unitary transformation which transfers information on initial conditions, contained in the usual Heisenberg-picture state vector, into the operators, leaving the state vector with no information whatsoever.

However, the only systems which are analyzed in [24] are ones in which the operators act only on qubits, i.e., vectors in a two-dimensional Hilbert space. No operators or parameters corresponding to location in space appear in the formalism. So, transfer of information from one place to another cannot be described within the mathematical formalism, and can only be dealt with at the level of verbal description. To demonstrate that the physical system in the EPRB or similar experiments² is local it is necessary to introduce into the formalism, in addition to the qubit operators, spatial degrees of freedom.

One way to accomplish this is to include, in addition to qubit operators, operators corresponding to spatial location. In [27] I constructed, in the Heisenberg picture, such a first-quantized model, applying it to an examination of Stapp’s claim [28] of a “core basis problem” in Everett quantum theory with spatial degrees of freedom. However, in this formalism the qubit operators still do not possess a location—that is to say, they are not parameterized by position in space, since position is an operator and cannot be

²In addition to the EPRB experiment, Deutsch and Hayden [24] apply their formalism to the phenomenon of teleportation. Hewitt-Horsman and Vedral have applied it to entanglement swapping [25] and multipartite entanglement [26].

used to parameterize another operator. Questions regarding location in space can only be addressed in an indirect manner (examples may be found in [27]). The first-quantized approach therefore does not seem to be the most efficient formalism to use in investigating the issue of locality.

The most natural way to introduce spatial degrees of freedom in a local manner into a quantum theory is to make it into a field theory. Indeed this is essentially the *only* way to construct a Lorentz-invariant quantum theory [29]. (Note, however, that we will only be concerned with nonrelativistic theories in this paper.) In [30] I analyzed a nonrelativistic quantum field theory of interacting spin-1/2 fermions with local interactions. I presented an explicit form for the transformation to the Deutsch-Hayden picture, for initial conditions corresponding to the generation of entanglement, and computed, to lowest order in perturbation theory, the generation of entanglement between pairs of particles³.

Can a model such as that in [30], a quantum field theory along with a way of transforming to the Deutsch-Hayden picture, be considered local in the sense of the definition above (Sec. 1.1) without qualification? This definition can be decomposed into four requirements, three of which are satisfied by the model of [30]:

- L1** A local encoding of the information which controls the probabilities of measurement outcomes. (The model of [30] satisfies this by virtue of the connection to the Deutsch-Hayden picture.⁴)
- L2** Local propagation of the information. (The model of [30] satisfies this because it relates operators at one spacetime point to those at another by means of local partial differential equations.)
- L3** Local interactions, at least on relevant scales. (The model of [30] satisfies this because the Hamiltonian is constructed of sums of products of finite numbers of field operators, or their derivatives, at the same spacetime point).

L3 might be considered to already be implied by L2. But, one could construct a theory with the usual kinetic terms, hence the usual propagators, but with nonlocal interaction terms, and such a theory would in general have nonlocality even when viewed as a classical theory. Indeed the model I will use in this paper has interactions, different from those in [30], which *are*, strictly speaking, nonlocal; however, the range of the nonlocality is limited—note the caveat following the comma in L3!

What the model of [30] lacks is

³Deutsch [31] has proposed an unconventional type of quantum field theory he terms a “qubit field theory.” This theory differs from usual quantum field theories (see, e.g., [32]) in that, e.g., field operators at different locations at the same time do not necessarily commute. The quantum field theories of [30] and of the present paper are completely conventional in their mathematical formalism.

⁴The specific transformation given in [30] will itself in general introduce nonlocality. However, this problem can be remedied using the new type of transformation given in this paper. See Section 3.1.

L4 Local representations of the observers⁵ and their states of awareness.

That is, the mathematical formalism must be able to tell us that Alice is *here* when she makes her spin measurement, that Bob is *there* when he makes his, and that Corry is *somewhere* when she receives the reports of Alice’s and Bob’s results. *Here*, *there* and *somewhere* need not be mathematical points, but must extend over regions which are small compared to, say, the distance between Alice and Bob when they make their measurements (assumed, as usual in such discussions, to be performed closely-enough to simultaneously that no material object could be present at both). One cannot examine what the formalism says about information moving from *here* to *there* unless the formalism says at least roughly where *here* and *there* are, and enables us to describe situations in which *here* is almost certainly distant from *there*⁶.

Furthermore, when the formalism determines whether, say, Alice is or is not *here*, it must do so using mathematical ingredients which are also *here*, i.e., in some way associated with *here*⁷. There may exist theories which specify that observers are at certain locations using mathematical objects which are far from those locations, or which are not associated with locations in space at all, but we will not consider those to be local unless they can be reformulated to meet the above criterion.

1.3 Aims of the present paper

The main goal of this paper, then, is to extend the model of [30] to include explicit, local representations of observers. Doing this will require adding to the model operator-valued fields corresponding to the presence of observers, in addition to the fields corresponding to

⁵The term “observer” refers to any system which records the results of measurements, e.g. a computer system, not necessarily a living being.

⁶ Lange [33][p. 3], interested in closeness rather than distance, puts it this way: “Can there be space or time separating a cause from its direct effects, or must a cause be local to its effects? I will presume that this question makes sense. But it makes sense only if a cause and its effects have locations in space and time. Otherwise, we can’t ask whether they must be near each other.”

⁷ Hardy [34] has termed this notion “F-locality:”

... [We] insist that, in making predictions for [space-time region] R , we only refer to mathematical objects pertaining to R (for, if not, what do we consider). This seems like a useful idea and deserves a name — we will call it formalism locality (or F-locality).

F-locality: A formulation of a physical theory is F-local if, in using it to make statements [in valid cases, validity being determined by the theory itself] about an arbitrary spacetime region R , we need only refer to mathematical objects pertaining to R .

Interestingly, Hardy does *not* consider local field theories to be F-local because, in going from the differential equations to the propagator, one must take into account boundary conditions specified on a region outside of R .

the spin-1/2 systems which the observers measure. It will also require an interpretational rule that will enable us to determine the probability that an observer is in a given region of space at a given time with a given state of awareness.

The interactions in the model will have to be such as to lead to transformation of the fields and resulting observer states of awareness corresponding to ideal measurements [35]. Quantum field theories are typically solved using perturbation theory, i.e., first expanding the quantity one wishes to compute in a power series in terms of a parameter related to the strength of the interaction between fields and then computing the lowest order terms in the expansion. But, measurement is fundamentally a nonperturbative process. When a measuring device in a “ready state” is presented with a to-be-measured system in state “1,” the measuring device should end up in a state “system observed in state 1,” not in a superposition of mostly “ready state” with a small admixture of “system observed in state 1.” So in analyzing the model it will be necessary to employ other, nonperturbative techniques to the extent possible.

The claim of locality depends critically on the possibility of transforming from the usual Heisenberg picture to the Deutsch-Hayden picture. I will therefore present explicit transformation rules for the EPRB scenario, and examine the issue of whether the field-theoretic Deutsch-Hayden transformation actually *introduces* a degree of nonlocality into the theory.

Let me point out three issues that will *not* be dealt with in this paper.

While the locality of the model depends on the existence of a transformation to the Deutsch-Hayden picture, all calculations will actually be carried out in the usual Heisenberg picture. Matrix elements are the same in both pictures, as are the equations of motion [30]. Undoubtedly computation done directly using operators in the Deutsch-Hayden picture can prove useful in quantum field theory as it has in quantum mechanics [24]–[26]. But it is not a logical necessity for demonstrating locality; the simple possibility of transforming to the Deutsch-Hayden picture suffices [30].

I will consider measured systems and observers in states such that they are well-localized on the scale of their separation from one another, and compute Bell-type correlations which bear on the issues of common-cause locality outlined above. Stapp [28] has raised an issue, which he terms the “core basis problem,” related to questions, not of locality, but of *localization*, its origin and persistence. How can the tendency of quantum-mechanical wave packets to spread out, and the lack of explicit wavefunction collapse in Everett quantum theory, be reconciled with the fact that observers perceive macroscopic objects—including themselves—to be at well-defined locations in space? I have discussed this issue elsewhere [27] in the context of first-quantized theory (see also [36]). Here I only wish to emphasize that this localization issue is *distinct* from the locality issue which is the focus of the present paper. As discussed above, if a theory could not in some way account for at least the perception of localization it would be impossible to talk about issues of locality. But even in the case that the localization problem is solved, or considered not to be a problem,

the question of locality remains to be addressed.

Finally, the question of the meaning of probability in Everett quantum theory remains a contentious one; see, e.g., [20], [50], [54] and references therein. This is clearly an important issue, but again one which is distinct from the locality issue which is of concern here.

1.4 Organization of the paper

Section 2 describes the features of the model, including the form of the interaction Hamiltonians, the interpretational rule, the initial state and approximation techniques. Section 3 deals with the transformation to the Deutsch-Hayden picture. Section 4 sets up the analysis of the EPRB experiment using the formalism of the model and computes the spin-measurement probabilities and correlation to all orders in the strength of the coupling between the spin-1/2 systems and the spin-measuring observers, and the correlation to lowest order in the strength of the coupling to the observer determining the correlations. Section 5 presents a summary and discussion.

2 Field-theoretic measurement model and approximation techniques

2.1 The model

2.1.1 Fields and free Hamiltonians

The fields and free Hamiltonians of the model are just the standard building blocks of nonrelativistic quantum field theory. The fields create observed systems and observers. In order to be applicable to the EPRB experiment, the model must contain

$$\text{Systems, } \mathcal{S}[p]: \quad \hat{\phi}_{[p]i}(\vec{x}, t), \quad p = 1, 2, \quad i = 1, 2 \quad (1)$$

$$\text{Observers, } \mathcal{O}[p]: \quad \hat{\chi}_{[p]i}(\vec{x}, t), \quad p = 1, 2, \quad i = 0, 1 \quad (2)$$

$$\text{Comparator, } \mathcal{C}: \quad \hat{\xi}_i(\vec{x}, t), \quad i = 0, 1 \quad (3)$$

Here

$$\vec{x} = (x_1, x_2, x_3) \quad (4)$$

is the spatial position and t is the time. The fields are Heisenberg-picture operators; they are equal to their Schrödinger-picture counterparts, which will be denoted by the same symbol without the time argument, at $t = 0$:

$$\hat{\phi}_{[p]i}(\vec{x}) = \hat{\phi}_{[p]i}(\vec{x}, 0), \quad \hat{\chi}_{[p]i}(\vec{x}) = \hat{\chi}_{[p]i}(\vec{x}, 0), \quad \hat{\xi}_i(\vec{x}) = \hat{\xi}_i(\vec{x}, 0). \quad (5)$$

The EPRB experiment involves the measurement of the spins of two spin-1/2 systems. We will take the systems to be quanta of different species, and will use an index in square brackets $[p]$, $p = 1, 2$ to label the two species⁸. The index i in (1) is the two-component spinor index, with $i = 1$ and $i = 2$ corresponding respectively to spin-up and spin down along the x_3 direction.

In the course of the EPRB experiment each spin-1/2 system will be measured by a distinct observer/measuring apparatus, and we will use the index $[p]$ to label the field $\hat{\chi}_{[p]i}(\vec{x}, t)$ corresponding to the observer who measures system $[p]$. Observers, whether sentient beings or simple machines, are composites of large numbers of elementary quanta, so the fields (2) that create them are to be viewed as effective composite operators [38]-[49], and it is certainly justified to model the observers as distinguishable—the index $[p]$ will be referred to as the “species” index even when it labels the two different observers.

Each observer can be in one of two internal states labeled by the index i in (2), $i = 0$ corresponding to a ready state or state of ignorance, and $i = 1$ corresponding to the state in which the observer has detected the system it measures to be in a spin-up state along some axis (the particular axis being determined by the choice of interaction Hamiltonian). That is, the observers do not determine the spin state of the system they measure, but rather detect the presence of a system in a particular spin state.

Subsequently, a third observer compares the results obtained by the two observers who directly measured the spin-1/2 particles. This “comparator” observer, corresponding to the field $\hat{\xi}_i(\vec{x})$, also can be in one of two internal states, with, again, $i = 0$ corresponding to a ready state or state of ignorance. In the presence of observers both of which are in the internal state 1 the comparator transitions to its internal state 1.

As a shorthand for “the system $[p]$,” “the observer $[p]$ ” and “the comparator,” the notations $\mathcal{S}[p]$, $\mathcal{O}[p]$ and \mathcal{C} will frequently be used. In scenarios involving only a single system and observer, we will simply refer to \mathcal{S} and \mathcal{O} .

All the fields (1)-(3) will be taken to be fermionic, obeying the usual equal-time anti-commutation relations

$$\begin{aligned}\{\hat{\phi}_{[p]i}(\vec{x}), \hat{\phi}_{[q]j}^\dagger(\vec{y})\} &= \delta_{pq}\delta_{ij}\delta^3(\vec{x} - \vec{y}), \quad p, q = 1, 2, \quad i, j = 1, 2, \\ \{\hat{\chi}_{[p]i}(\vec{x}), \hat{\chi}_{[q]j}^\dagger(\vec{y})\} &= \delta_{pq}\delta_{ij}\delta^3(\vec{x} - \vec{y}), \quad p, q = 1, 2, \quad i, j = 0, 1, \\ \{\hat{\xi}_i(\vec{x}), \hat{\xi}_j^\dagger(\vec{y})\} &= \delta_{ij}\delta^3(\vec{x} - \vec{y}), \quad i, j = 0, 1,\end{aligned}\tag{6}$$

⁸Tests of Bell’s theorem tend to use pairs of identical particles such as photons (see, e.g., [55], [56]), but in the experimental configurations the two particles are effectively distinguishable by virtue of their spatial separation. In any case, the issues involved in Bell’s theorem and the EPRB experiment are indifferent as to whether the particles involved are distinguishable quanta of different fields or indistinguishable quanta of the same field, so here as in [30] I make the simplifying choice of different species for the measured systems.

with all other anticommutators vanishing,

$$\{\hat{\phi}_{[p]i}(\vec{x}), \hat{\phi}_{[q]j}(\vec{y})\} = 0, \{\hat{\chi}_{[p]i}^\dagger(\vec{x}), \hat{\chi}_{[q]j}^\dagger(\vec{y})\} = 0, \{\hat{\xi}_i(\vec{x}), \hat{\chi}_{[p]j}^\dagger(\vec{y})\} = 0, \text{ etc.} \quad (7)$$

The vacuum state $|0\rangle$ is annihilated by the fields:

$$\begin{aligned} \hat{\phi}_{[p]i}(\vec{x})|0\rangle &= 0, & p = 1, 2, \quad i = 1, 2, \\ \hat{\chi}_{[p]i}(\vec{x})|0\rangle &= 0, & p = 1, 2, \quad i = 0, 1, \\ \hat{\xi}_i(\vec{x})|0\rangle &= 0, & i = 0, 1, \end{aligned} \quad (8)$$

Since the $\mathcal{O}[p]$'s will be measuring the spins of the $\mathcal{S}[p]$'s along arbitrary axes, not just the x_3 direction, we will need expressions for field operators along an axis

$$\vec{n} = (\sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\phi)). \quad (9)$$

The required relations are [63]

$$\hat{\phi}_{\vec{n},[p],1}^\dagger(\vec{x}) = e^{-i\phi/2} \cos\left(\frac{\theta}{2}\right) \hat{\phi}_{[p]1}^\dagger(\vec{x}) + e^{i\phi/2} \sin\left(\frac{\theta}{2}\right) \hat{\phi}_{[p]2}^\dagger(\vec{x}) \quad (10)$$

$$\hat{\phi}_{\vec{n},[p],2}^\dagger(\vec{x}) = -e^{-i\phi/2} \sin\left(\frac{\theta}{2}\right) \hat{\phi}_{[p]1}^\dagger(\vec{x}) + e^{i\phi/2} \cos\left(\frac{\theta}{2}\right) \hat{\phi}_{[p]2}^\dagger(\vec{x}) \quad (11)$$

and their adjoints.

The free Hamiltonians for the fields are

$$\widehat{H}_F^{\mathcal{S}[p]} = \hbar^2 \sum_{i=1,2} \int d^3\vec{x} \hat{\phi}_{[p]i}^\dagger(\vec{x}) \left(-\frac{\vec{\nabla}^2}{2m_{\mathcal{S}[p]}} \right) \hat{\phi}_{[p]i}(\vec{x}), \quad p = 1, 2, \quad (12)$$

$$\widehat{H}_F^{\mathcal{O}[p]} = \hbar^2 \sum_{i=0,1} \int d^3\vec{x} \hat{\chi}_{[p]i}^\dagger(\vec{x}) \left(-\frac{\vec{\nabla}^2}{2m_{\mathcal{O}[p]}} \right) \hat{\chi}_{[p]i}(\vec{x}), \quad p = 1, 2, \quad (13)$$

$$\widehat{H}_F^{\mathcal{C}} = \hbar^2 \sum_{i=0,1} \int d^3\vec{x} \hat{\xi}_i^\dagger(\vec{x}) \left(-\frac{\vec{\nabla}^2}{2m_{\mathcal{C}}} \right) \hat{\xi}_i(\vec{x}), \quad (14)$$

with $m_{\mathcal{S}[p]}$, $m_{\mathcal{O}[p]}$ and $m_{\mathcal{C}}$ the masses of the $\mathcal{S}[p]$, $\mathcal{O}[p]$ and \mathcal{C} respectively.

2.1.2 Finite-range interactions

All of the interactions in the model are field-theoretic versions of the type of interaction used in [27], an instantaneous interaction-at-a-distance of limited range. Consider first the interaction Hamiltonian responsible for measurement of $\mathcal{S}[p]$ by $\mathcal{O}[p]$. As will be seen in Section 4 this causes $\mathcal{O}[p]$ to transition out of the ready state 0 into the “system-detected”

state 1 in the presence of $\mathcal{S}[p]$, provided $\mathcal{S}[p]$ is polarized parallel to the axis of $\mathcal{O}[p]$'s spin analyzer.

$$\widehat{H}_M^{\mathcal{OS}[p]} = \int d^3\vec{x} d^3\vec{y} \widehat{\mathcal{H}}_M^{\mathcal{OS}[p]}(\vec{x}, \vec{y}), \quad (15)$$

where

$$\widehat{\mathcal{H}}_M^{\mathcal{OS}[p]}(\vec{x}, \vec{y}) = \widehat{h}_M^{\mathcal{O}[p]}(\vec{x}) f_{[p]}(\vec{x}, \vec{y}) \widehat{\mathcal{N}}_{\vec{n}[p],1}^{\mathcal{S}[p]}(\vec{y}), \quad p = 1, 2, \quad (16)$$

with

$$\widehat{h}_M^{\mathcal{O}[p]}(\vec{x}) = i\kappa \left(\widehat{\chi}_{[p]1}^\dagger(\vec{x}) \widehat{\chi}_{[p]0}(\vec{x}) - \widehat{\chi}_{[p]0}^\dagger(\vec{x}) \widehat{\chi}_{[p]1}(\vec{x}) \right), \quad p = 1, 2, \quad (17)$$

$$\widehat{\mathcal{N}}_{\vec{n}[p],i}^{\mathcal{S}[p]}(\vec{x}) = \widehat{\phi}_{\vec{n}[p],[p],i}^\dagger(\vec{x}) \widehat{\phi}_{\vec{n}[p],[p],i}(\vec{x}), \quad p = 1, 2, \quad i = 1, 2, \quad (18)$$

and

$$f_{[p]}(\vec{x}, \vec{y}) = \theta(a_{[p]} - |\vec{x} - \vec{y}|), \quad a_{[p]} > 0, \quad p = 1, 2. \quad (19)$$

$\widehat{\mathcal{N}}_{\vec{n}[p],1}^{\mathcal{S}[p]}(\vec{x})$ ($\widehat{\mathcal{N}}_{\vec{n}[p],2}^{\mathcal{S}[p]}(\vec{x})$) is the number density operator for $\mathcal{S}[p]$ polarized spin-up (spin-down) along the axis $\vec{n}[p]$ of $\mathcal{O}[p]$'s spin analyzer. $\theta(x)$ is the Heaviside step function.

As will be seen, $\widehat{h}_M^{\mathcal{O}[p]}(\vec{x})$ drives transitions between states 0 and 1 of $\mathcal{O}[p]$. Clearly (16) involves an instantaneous interaction-at-a-distance. What renders the interaction Hamiltonian (15) suitable for use in a model demonstrating locality is the function $f_{[p]}(\vec{x}, \vec{y})$, which forces the interaction to vanish for distances larger than $a_{[p]}$. This is then an effectively local interaction, suitable for examining the EPRB scenario, provided the respective locations at which $\mathcal{S}[1]$ is measured by $\mathcal{O}[1]$ and that at which $\mathcal{S}[2]$ is measured by $\mathcal{O}[2]$ are separated by a distance much larger than $a_{[1]}$ and $a_{[2]}$.

A similar interaction is used for the measurement of $\mathcal{O}[1]$ and $\mathcal{O}[2]$ by \mathcal{C} :

$$\widehat{H}_M^{\mathcal{CO}} = \int d^3\vec{x} d^3\vec{y} d^3\vec{z} \widehat{\mathcal{H}}_M^{\mathcal{CO}}(\vec{x}, \vec{y}, \vec{z}), \quad (20)$$

where

$$\widehat{\mathcal{H}}_M^{\mathcal{CO}}(\vec{x}, \vec{y}, \vec{z}) = \widehat{h}_M^{\mathcal{C}}(\vec{x}) f^{\mathcal{C}}(\vec{x}, \vec{y}) f^{\mathcal{C}}(\vec{x}, \vec{z}) \widehat{\mathcal{N}}_1^{\mathcal{O}[1]}(\vec{y}) \widehat{\mathcal{N}}_1^{\mathcal{O}[2]}(\vec{z}), \quad (21)$$

with

$$\widehat{h}_M^{\mathcal{C}}(\vec{x}) = i\kappa_{\mathcal{C}} \left(\widehat{\xi}_1^\dagger(\vec{x}) \widehat{\xi}_0(\vec{x}) - \widehat{\xi}_0^\dagger(\vec{x}) \widehat{\xi}_1(\vec{x}) \right), \quad (22)$$

$$\widehat{\mathcal{N}}_i^{\mathcal{O}[p]}(\vec{x}) = \widehat{\chi}_{[p]i}^\dagger(\vec{x}) \widehat{\chi}_{[p]i}(\vec{x}), \quad p = 1, 2, \quad i = 0, 1, \quad (23)$$

and

$$f_{\mathcal{C}}(\vec{x}, \vec{y}) = \theta(a_{\mathcal{C}} - |\vec{x} - \vec{y}|), \quad a_{\mathcal{C}} > 0. \quad (24)$$

2.1.3 Interpretational rule

Ideal measurement in quantum mechanics is usually described in terms of the eigenvalue-eigenstate link [50]. In Everett quantum mechanics in the Heisenberg picture, this can be expressed in terms of transformations of the operator representing the state of awareness of the observer in the course of the measurement interaction [27], [51]-[53]. Scattering states in field theory are also described as eigenstates of projection operators [32]. In all of these cases probabilities are matrix elements of projection operators.

It is not clear how or if it is possible in general to represent a property such as “an observer localized in such-and-such a region has detected a system which is spin-up” within this framework. *At least for the limited purposes of the present model* the following rule, which does not make use of the eigenvalue-eigenstate link, is adequate:

Interpretational rule: If for some spatial region Ω

$$\int_{\Omega} d^3\vec{x} \langle \psi_{in} | \widehat{\mathcal{N}}_i^{\mathcal{O}[p]}(\vec{x}, t) | \psi_{in} \rangle > 0, \quad (25)$$

where $|\psi_{in}\rangle$ is the Heisenberg-picture initial state, then an observer $\mathcal{O}[p]$ in state of awareness i exists at time t . The observer is located in the smallest Ω for which (25) holds. The probability associated with the observer-in-state-of-awareness i is

$$P_i^{[p]}(t) = \int_{\Omega} d^3\vec{x} \langle \psi_{in} | \widehat{\mathcal{N}}_i^{\mathcal{O}[p]}(\vec{x}, t) | \psi_{in} \rangle. \quad (26)$$

A similar rule, of course, also applies for \mathcal{C} .

This simple rule⁹ is adequate only by virtue of the conditions placed on the model’s initial state (Section 2.1.4) and the massive narrow-wavepacket limit (Section 2.2.2).

2.1.4 Initial state

S1: The Heisenberg-picture initial state is such that there is only a single quantum of each species present.

This should not strike the reader as problematic. As already discussed, I have chosen the two measured systems to be quanta of different species; given that, saying that these are the only ones present during the experiment is simply assuming that the experiment is run properly. Since we are dealing with a nonrelativistic theory we need not consider new quanta coming into being within the region in which the experiment is taking place. As for the observers $\mathcal{O}[1]$, $\mathcal{O}[2]$ and \mathcal{C} , they are macroscopic completely-distinguishable systems of which there certainly will be only one of each “species” present.

⁹For other definitions of localization in quantum field theory see [57], [58].

Condition S1 ensures that the number-density operator will measure the probability of a single observer being at some location; e.g., a large value for $\widehat{\mathcal{N}}_i^{\mathcal{O}[p]}(\vec{x})$ increases the likelihood that $\mathcal{O}[p]$ with awareness i is in a region containing \vec{x} .

S2: At the initial time each observer is well-localized in a region well-separated from the other observers and the systems to be measured.

S3 At the initial time each observer is definitely in a state of ignorance.

These are to make sure we are indeed modeling the locality-testing situations we are interested in. For mathematical convenience we will implement localization with Gaussian wavepackets. The EPRB initial state is

$$\begin{aligned} |\psi_{in}^E\rangle &= \frac{1}{\sqrt{2}} \int d^3x d^3y d^3z d^3v d^3w \psi_g^{\mathcal{C}}(\vec{x}) \psi_g^{\mathcal{O}[1]}(\vec{y}) \psi_g^{\mathcal{O}[2]}(\vec{z}) \psi_g^{\mathcal{S}[1]}(\vec{v}) \psi_g^{\mathcal{S}[2]}(\vec{w}) \\ &\quad \hat{\xi}_0^\dagger(\vec{x}) \hat{\chi}_{[1]0}^\dagger(\vec{y}) \hat{\chi}_{[2]0}^\dagger(\vec{z}) \left(\hat{\phi}_{[1]1}^\dagger(\vec{v}) \hat{\phi}_{[2]2}^\dagger(\vec{w}) - \hat{\phi}_{[1]2}^\dagger(\vec{v}) \hat{\phi}_{[2]1}^\dagger(\vec{w}) \right) |0\rangle. \end{aligned} \quad (27)$$

Here $\psi_g^{\mathcal{S}[p]}(\vec{x})$, $\psi_g^{\mathcal{O}[p]}(\vec{x})$ and $\psi_g^{\mathcal{C}}(\vec{x})$ are Gaussian wavepackets centered at locations $\vec{x}_{\mathcal{S}[p]}$, $\vec{x}_{\mathcal{O}[p]}$ and $\vec{x}_{\mathcal{C}}$ with widths $(\alpha_{\mathcal{S}[p]})^{-1/2}$, $(\alpha_{\mathcal{O}[p]})^{-1/2}$ and $(\alpha_{\mathcal{C}})^{-1/2}$, respectively :

$$\psi_g^{\mathcal{S}[p]}(\vec{x}) = \left(\frac{\alpha_{\mathcal{S}[p]}}{\pi} \right)^{3/4} \exp \left(-\frac{\alpha_{\mathcal{S}[p]} |\vec{x} - \vec{x}_{\mathcal{S}[p]}|^2}{2} + \frac{im_{\mathcal{S}[p]} \vec{v}_{\mathcal{S}[p]} \cdot (\vec{x} - \vec{x}_{\mathcal{S}[p]})}{\hbar} \right), \quad p = 1, 2 \quad (28)$$

$$\psi_g^{\mathcal{O}[p]}(\vec{x}) = \left(\frac{\alpha_{\mathcal{O}[p]}}{\pi} \right)^{3/4} \exp \left(-\frac{\alpha_{\mathcal{O}[p]} |\vec{x} - \vec{x}_{\mathcal{O}[p]}|^2}{2} + \frac{im_{\mathcal{O}[p]} \vec{v}_{\mathcal{O}[p]} \cdot (\vec{x} - \vec{x}_{\mathcal{O}[p]})}{\hbar} \right), \quad p = 1, 2 \quad (29)$$

$$\psi_g^{\mathcal{C}}(\vec{x}) = \left(\frac{\alpha_{\mathcal{C}}}{\pi} \right)^{3/4} \exp \left(-\frac{\alpha_{\mathcal{C}} |\vec{x} - \vec{x}_{\mathcal{C}}|^2}{2} + \frac{im_{\mathcal{C}} \vec{v}_{\mathcal{C}} \cdot (\vec{x} - \vec{x}_{\mathcal{C}})}{\hbar} \right) \quad (30)$$

The imaginary phases in (28)-(30) correspond to free motion of the wavepackets with velocities $\vec{v}_{\mathcal{S}[p]}$, $\vec{v}_{\mathcal{O}[p]}$ and $\vec{v}_{\mathcal{C}}$, respectively.

S4 The initial conditions are such that the free motion of the systems and observers will bring them into proximity with each other only for limited periods of time.

This is to allow the use of the sudden approximation.

2.2 Approximation techniques

2.2.1 Sudden approximation

To solve the model we employ a version of the sudden approximation [22]. While the systems are far from the observers, we will ignore the interaction terms in the complete Hamiltonian. Conversely, for the brief periods during which they are near each other we will ignore the kinetic terms. We will then take the limit in which the strength of the interaction becomes infinite and the time during which the interaction takes place goes to zero.

Specifically, the initial locations and velocities of the wavepackets for the $\mathcal{S}[p]$, $\mathcal{O}[p]$ and \mathcal{C} are chosen so that we can apply the following approximations during successive time intervals:

$$t_0 \leq t \leq t_1$$

t_0 is the initial time at which the Heisenberg-picture state vector is defined. At this time $\mathcal{O}[1]$, $\mathcal{O}[2]$, and \mathcal{C} are in well-separated Gaussian wavepackets. $\mathcal{S}[1]$ and $\mathcal{S}[2]$ are in a spin-entangled singlet state, each in Gaussian wavepackets coincident in location but with different velocities (see (27)). The initial positions and velocities of $\mathcal{S}[p]$ and $\mathcal{O}[p]$ are chosen so that they remain well-separated until time t_1 . So, during the time interval from t_0 to t_1 we will ignore the interaction terms and approximate the total Hamiltonian \widehat{H} by

$$\widehat{H} \approx \widehat{H}_{[0,1]} = \widehat{H}_F \quad (31)$$

where the total free Hamiltonian is

$$\widehat{H}_F = \sum_{p=1}^2 \left(\widehat{H}_F^{\mathcal{S}[p]} + \widehat{H}_F^{\mathcal{O}[p]} \right) + \widehat{H}_F^{\mathcal{C}}. \quad (32)$$

$$t_1 \leq t \leq t_2$$

$\mathcal{S}[p]$ and $\mathcal{O}[p]$ are within $a_{[p]}$ of each other, and we ignore the free Hamiltonians for them. (For simplicity $\mathcal{S}[1]$ and $\mathcal{O}[1]$ are taken to be close to one another at the same time that $\mathcal{S}[2]$ and $\mathcal{O}[2]$ are close.)

$$\widehat{H} \approx \widehat{H}_{[1,2]} = \sum_{p=1}^2 \widehat{H}_M^{\mathcal{OS}[p]} + \widehat{H}_F^{\mathcal{C}}. \quad (33)$$

$$t_2 \leq t \leq t_3$$

All of the $\mathcal{S}[p]$, $\mathcal{O}[p]$, and \mathcal{C} are far enough from each other that we can ignore the interaction terms:

$$\widehat{H} \approx \widehat{H}_{[2,3]} = \widehat{H}_F. \quad (34)$$

$$t_3 \leq t \leq t_4$$

$\mathcal{O}[1]$ and $\mathcal{O}[2]$ are both within a_C of \mathcal{C} , so we ignore their free Hamiltonians and that of \mathcal{C} :

$$\widehat{H} \approx \widehat{H}_{[3,4]} = \sum_{p=1}^2 \widehat{H}_F^{\mathcal{S}[p]} + \widehat{H}_M^{\mathcal{CO}}. \quad (35)$$

$$t_4 \leq t \leq t_5$$

All of the $\mathcal{S}[p]$, $\mathcal{O}[p]$, and \mathcal{C} are again far enough from each other that we can ignore the interaction terms:

$$\widehat{H} \approx \widehat{H}_{[4,5]} = \widehat{H}_F. \quad (36)$$

Above and throughout t_5 denotes any time after t_4 . To indicate a time which is within one of the time windows above but is otherwise unspecified the notation $t_{[n-1,n]}$ will be used; i.e.,

$$t = t_{[n-1,n]} \Rightarrow t_{n-1} \leq t \leq t_n \quad (37)$$

For any operator the Schrödinger picture and Heisenberg picture are related by

$$\widehat{A}(t) = \widehat{U}^\dagger(t) \widehat{A} \widehat{U}(t), \quad (38)$$

where $\widehat{U}(t)$ is the unitary operator that generates time evolution between the initial time t_0 and time t . The two pictures are identical at $t = t_0$, so

$$\widehat{U}(t_0) = 1. \quad (39)$$

Define

$$\widehat{U}_{[n-1,n]} = \exp \left[-i \left(\frac{t_n - t_{n-1}}{\hbar} \right) \widehat{H}_{[n-1,n]} \right] \quad (40)$$

and

$$\widehat{U}_{[n-1,n]}(t) = \exp \left[-i \left(\frac{t - t_{n-1}}{\hbar} \right) \widehat{H}_{[n-1,n]} \right], \quad t_{n-1} \leq t \leq t_n \quad (41)$$

or, using (37),

$$\widehat{U}_{[n-1,n]}(t_{[n-1,n]}) = \exp \left[-i \left(\frac{t_{[n-1,n]} - t_{n-1}}{\hbar} \right) \widehat{H}_{[n-1,n]} \right]. \quad (42)$$

Using (31), (33)-(36), (40) and (42),

$$\widehat{U}(t_{[n-1,n]}) = \widehat{U}_{[n-1,n]}(t_{[n-1,n]}) \widehat{U}_{[n-2,n-1]} \widehat{U}_{[n-3,n-2]} \cdots \widehat{U}_{[1,2]} \widehat{U}_{[0,1]}. \quad (43)$$

In implementing the large-interaction-strength/short-interaction-time limit, we will take

$$\lim \kappa = \infty, \quad \lim(t_2 - t_1) = 0 \quad \text{s. t.} \quad \lim \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right) = \frac{\pi}{2}, \quad (44)$$

$$\lim \kappa_{\mathcal{C}} = \infty, \quad \lim(t_4 - t_3) = 0 \quad \text{s. t.} \quad \lim \left(\frac{\kappa_{\mathcal{C}}(t_4 - t_3)}{\hbar} \right) = \frac{\pi}{2}. \quad (45)$$

In addition, we will assume that the experiment has been “perfectly aligned,” in the sense that the initial positions and velocities of the wavepackets have been chosen so that the center of the $\mathcal{S}[p]$ wavepacket at time t_1 is at precisely the same location as that of $\mathcal{O}[p]$ for $p = 1, 2$, and those of $\mathcal{O}[1]$ and $\mathcal{O}[2]$ coincide with \mathcal{C} at t_3 :

$$\vec{x}_{\mathcal{S}[p]}(t_1) = \vec{x}_{\mathcal{O}[p]}(t_1), \quad p = 1, 2, \quad (46)$$

$$\vec{x}_{\mathcal{O}[1]}(t_3) = \vec{x}_{\mathcal{O}[2]}(t_3) = \vec{x}_{\mathcal{C}}(t_3), \quad (47)$$

where

$$\begin{aligned} \vec{x}_{\mathcal{S}[p]}(t) &= \vec{x}_{\mathcal{S}[p]} + \vec{v}_{\mathcal{S}[p]}(t - t_0), & p = 1, 2, \\ \vec{x}_{\mathcal{O}[p]}(t) &= \vec{x}_{\mathcal{O}[p]} + \vec{v}_{\mathcal{O}[p]}(t - t_0), & p = 1, 2, \\ \vec{x}_{\mathcal{C}}(t) &= \vec{x}_{\mathcal{C}} + \vec{v}_{\mathcal{C}}(t - t_0). \end{aligned} \quad (48)$$

In light of the massive narrow-wavepacket limit (Section 2.2.2), these conditions are stronger than necessary; it is only required that the encounters be to within $a_{[p]}$ or $a_{\mathcal{C}}$. However, the perfect-alignment conditions (46), (47) somewhat simplify the calculations.

2.2.2 Massive narrow-wavepacket limit

The observers are macroscopic instruments, and the aperture diameters $a_{[p]}$, $a_{\mathcal{C}}$ are macroscopic. Furthermore the measured systems involved in the measurement of spin are prepared in beams which with very high probability enter the required area of the experimental apparatus (else the experimental setup must be redesigned!) So, for purposes of mathematical convenience, we will consider the limit in which observers and systems are in *infinitely* narrow wavepackets. For such a wavepacket to persist in a well-localized state—and it must do so at least long enough for the experiment to be completed—we will also have to let the masses of the systems and observers become infinite. This limit will be referred to as the “massive narrow-wavepacket” or “MN” limit. Specifically,

$$\lim_{MN} \alpha = \infty, \quad \lim_{MN} m = 0 \quad \text{s. t.} \quad \lim_{MN} \left(\frac{\alpha \hbar \Delta t}{m} \right) = 0, \quad (49)$$

where $\alpha^{-1/2}$ is the width of the initial wavepacket for the field in question, m is the relevant mass (see (28)-(30)) and Δt is the relevant time scale over which spreading must be avoided (i.e., some time longer than $t_4 - t_0$).

Such a state of affairs, with all entities in precise spatial locations, may hardly seem like a *quantum*-mechanical system. But I emphasize again that Bell's theorem and the issue of locality has nothing to do with localization or its absence (i.e., quantum-mechanical spatial spreading), except to the extent that localization on a sufficiently large scale is necessary to talk about locality at all. The sceptical reader may examine any of the many derivations of the many versions of Bell's theorem.¹⁰

3 Transformation from the Heisenberg picture to the Deutsch-Hayden picture

The transformation from the Heisenberg picture to the Deutsch-Hayden picture, which will be referred to as the Deutsch-Hayden transformation, is a unitary transformation which maps the initial Heisenberg-picture state to a standard state which is independent of any information about the physical configuration of the system being described. In field theory a natural choice for this standard state is the vacuum state, and we will refer to the Deutsch-Hayden picture in which this choice is made as the vacuum representation [30].

A completely local transformation would be one in which a transformed field at \vec{x} is a function only of fields and wavefunctions—the wavefunctions appearing in the expression for the Heisenberg-picture state (27)—at \vec{x} . For our purposes it will be sufficient if the transformation is effectively local in the same spirit as the interaction Hamiltonian, i.e., a function only of fields and wavefunctions sufficiently close to \vec{x} .

It is possible to obtain such a transformation by introducing into the model fictitious static fields which contribute fictitious quanta to the initial state. There are no kinetic or interaction terms in the Hamiltonian for these fields, and their wavefunctions are purely arbitrary aside from normalization. They add one further aspect to the arbitrariness of the standard state vector in the Deutsch-Hayden picture. They clearly carry no physical information, and their presence in no way affects the characteristic of the Deutsch-Hayden picture that it removes all physical information from the state vector and places it into the field operators.

¹⁰Two particularly nice ones are [59], [60].

3.1 Deutsch-Hayden transformation: single system and observer

For clarity consider first a situation in which a single observer \mathcal{O} measures a system \mathcal{S} , with initial state

$$|\psi_{in}^{\mathcal{OS}}\rangle = \sum_{i=1}^2 b_i \int d^3\vec{x} d^3\vec{y} \psi_g^{\mathcal{O}}(\vec{x}) \psi_g^{\mathcal{S}}(\vec{y}) \hat{\chi}_0^\dagger(\vec{x}) \hat{\phi}_i^\dagger(\vec{y}) |0\rangle, \quad (50)$$

The fictitious fields $\hat{\zeta}_{\mathcal{S}}(\vec{x})$, $\hat{\zeta}_{\mathcal{O}}(\vec{x})$, satisfy the usual anticommutation relations with their adjoints,

$$\{\hat{\zeta}_{\mathcal{S}}(\vec{x}), \hat{\zeta}_{\mathcal{S}}^\dagger(\vec{y})\} = \delta^3(\vec{x} - \vec{y}), \quad (51)$$

and anticommute with each other and all other fields,

$$\{\hat{\zeta}_{\mathcal{S}}(\vec{x}), \hat{\zeta}_{\mathcal{S}}(\vec{y})\} = 0, \{\hat{\zeta}_{\mathcal{O}}(\vec{x}), \hat{\zeta}_{\mathcal{O}}(\vec{y})\} = 0, \{\hat{\zeta}_{\mathcal{S}}(\vec{x}), \hat{\zeta}_{\mathcal{S}}(\vec{y})\} = 0, \text{ etc..} \quad (52)$$

The modified initial state, including fictitious fields, is

$$|\psi_{in}^{\mathcal{OS}'}\rangle = \sum_{i=1}^2 b_i \int d^3\vec{x} d^3\vec{w} d^3\vec{y} d^3\vec{z} \psi_g^{\mathcal{O}}(\vec{x}) \psi^{\mathcal{O}'}(\vec{w}) \psi_g^{\mathcal{S}}(\vec{y}) \psi^{\mathcal{S}'}(\vec{z}) \hat{\chi}_0^\dagger(\vec{x}) \hat{\zeta}_{\mathcal{O}}^\dagger(\vec{w}) \hat{\phi}_i^\dagger(\vec{y}) \hat{\zeta}_{\mathcal{S}}^\dagger(\vec{z}) |0\rangle. \quad (53)$$

The wavefunctions for the fictitious fields are normalized,

$$\int d^3\vec{x} |\psi^{\mathcal{O}'}(\vec{x})|^2 = \int d^3\vec{x} |\psi^{\mathcal{S}'}(\vec{x})|^2 = 1, \quad (54)$$

but otherwise arbitrary. Clearly the expectation value of any function of the $\hat{\chi}_i^\dagger(\vec{x})$'s or the $\hat{\phi}_i^\dagger(\vec{x})$'s will be the same in the state (53) as in (50) regardless of the values of the fictitious-field wavefunctions.

Define the skew-Hermitian operators

$$\widehat{W}^{\mathcal{S}} = \sum_{i=1}^2 b_i \int d^3\vec{y} d^3\vec{z} \left(\psi_g^{\mathcal{S}}(\vec{y}) \psi^{\mathcal{S}'}(\vec{z}) \hat{\phi}_i^\dagger(\vec{y}) \hat{\zeta}_{\mathcal{S}}^\dagger(\vec{z}) - \psi_g^{\mathcal{S}*}(\vec{y}) \psi^{\mathcal{S}'*}(\vec{z}) \hat{\zeta}_{\mathcal{S}}(\vec{z}) \hat{\phi}_i(\vec{y}) \right) \quad (55)$$

and

$$\widehat{W}^{\mathcal{O}} = \int d^3\vec{y} d^3\vec{z} \left(\psi_g^{\mathcal{O}}(\vec{y}) \psi^{\mathcal{O}'}(\vec{z}) \hat{\chi}_0^\dagger(\vec{y}) \hat{\zeta}_{\mathcal{O}}^\dagger(\vec{z}) - \psi_g^{\mathcal{O}*}(\vec{y}) \psi^{\mathcal{O}'*}(\vec{z}) \hat{\zeta}_{\mathcal{O}}(\vec{z}) \hat{\chi}_0(\vec{y}) \right), \quad (56)$$

and the unitary operators

$$\widehat{V}^{\mathcal{S}} = \exp\left(\frac{\pi}{2} \widehat{W}^{\mathcal{S}}\right), \quad (57)$$

$$\widehat{V}^{\mathcal{O}} = \exp\left(\frac{\pi}{2} \widehat{W}^{\mathcal{O}}\right), \quad (58)$$

$$\widehat{V}^{\mathcal{OS}} = \widehat{V}^{\mathcal{O}} \widehat{V}^{\mathcal{S}}, \quad (59)$$

Using (6)-(8), (51)-(59) and mathematical induction, we find that $\hat{V}^{\mathcal{OS}\dagger}$ implements a Deutsch-Hayden transformation:

$$\hat{V}^{\mathcal{OS}\dagger}|\psi_{in}^{\mathcal{OS}'}\rangle = |0\rangle. \quad (60)$$

The corresponding unitary transformations of the operators are

$$\hat{\phi}_{V\mathcal{OS},i}(\vec{x}) = \hat{V}^{\mathcal{OS}\dagger} \hat{\phi}_i(\vec{x}) \hat{V}^{\mathcal{OS}}, \quad i = 1, 2 \quad (61)$$

$$\hat{\chi}_{V\mathcal{OS},i}(\vec{x}) = \hat{V}^{\mathcal{OS}\dagger} \hat{\chi}_i(\vec{x}) \hat{V}^{\mathcal{OS}}, \quad i = 0, 1 \quad (62)$$

Since $\widehat{W}^{\mathcal{S}}$ and $\widehat{W}^{\mathcal{O}}$ are bilinear in anticommuting fields,

$$[\widehat{W}^{\mathcal{S}}, \widehat{W}^{\mathcal{O}}] = 0, \quad (63)$$

$$[\widehat{W}^{\mathcal{S}}, \hat{\chi}_i(\vec{x})] = 0, \quad i = 1, 2 \quad (64)$$

and

$$[\widehat{W}^{\mathcal{O}}, \hat{\phi}_i(\vec{x})] = 0, \quad i = 1, 2 \quad (65)$$

Therefore

$$\hat{\phi}_{V\mathcal{OS},i}(\vec{x}) = \hat{V}^{\mathcal{S}\dagger} \hat{\phi}_i(\vec{x}) \hat{V}^{\mathcal{S}}, \quad i = 1, 2 \quad (66)$$

$$\hat{\chi}_{V\mathcal{OS},i}(\vec{x}) = \hat{V}^{\mathcal{O}\dagger} \hat{\chi}_i(\vec{x}) \hat{V}^{\mathcal{O}}, \quad i = 0, 1 \quad (67)$$

So, we can investigate the locality of the transformation of each field separately. Consider, e.g., (67). Using (6), (7), (51), (52), (56) and (58), the formula [61]

$$\exp(-yF)G\exp(yF) = G + y[G, F] + \frac{y^2}{2!}[[G, F], f] + \frac{y^3}{3!}[[[G, F]F,]F] + \dots, \quad (68)$$

and mathematical induction, we obtain

$$\hat{\chi}_{V\mathcal{OS},i}(\vec{x}) = \hat{\chi}_i(\vec{x}) + \delta_{i,0}\psi_g^{\mathcal{O}}(\vec{x}) \int d^3\vec{y} \left(\psi^{\mathcal{O}'}(\vec{y}) \hat{\zeta}_{\mathcal{O}}^{\dagger}(\vec{y}) - \psi_g^{\mathcal{O}*}(\vec{y}) \hat{\chi}_0(\vec{y}) \right) \quad (69)$$

To see that this is an effectively local transformation, consider the case $i = 0$ in (69). First suppose \vec{x} is far from the center $\vec{x}_{\mathcal{O}}$ of the \mathcal{O} wavepacket. Then, since $\psi_g^{\mathcal{O}}(\vec{x})$ is nearly zero, $\hat{\chi}_{V\mathcal{OS},i}(\vec{x})$ is nearly equal to $\hat{\chi}_i(\vec{x})$ and thus nearly independent of operators or wavefunctions at locations not close to \vec{x} . This will also be true if \vec{x} is close to $\vec{x}_{\mathcal{O}}$. The second term in the integrand in (69) is negligible unless \vec{y} is also close to $\vec{x}_{\mathcal{O}}$. The first term can be large for any value of \vec{y} , but this involves only the fictitious-field operator and wavefunction and so does not involve any encoding by the Deutsch-Hayden transformation of physical information into $\hat{\chi}_{V\mathcal{OS},i}(\vec{x})$ coming from a location \vec{y} far from \vec{x} . Indeed, since the only restriction on $\psi^{\mathcal{O}'}(\vec{x})$ is normalization, we could simply impose the requirement that $\psi^{\mathcal{O}'}(\vec{x})$ also be localized near $\vec{x}_{\mathcal{O}}$.

(The same sorts of considerations, applied to eqs. (150) and (151) of [30], show that the Deutsch-Hayden transformation I employed in that paper, lacking fictitious fields, is only local if $\psi_{[1]1}(\vec{x})$ and $\psi_{[2]2}(\vec{x})$ are both localized near the same point. This restriction can be removed by applying the present fictitious-field method to that case.)

3.2 Deutsch-Hayden transformation: EPRB

The modified initial state vector with fictitious fields is

$$\begin{aligned}
|\psi_{in}^{E'}\rangle &= \frac{1}{\sqrt{2}} \int d^3\vec{x} d^3\vec{y} d^3\vec{v} d^3\vec{r} d^3\vec{s} d^3\vec{z}_C d^3\vec{z}_{O1} d^3\vec{z}_{O2} d^3\vec{z}_{S1} d^3\vec{z}_{S2} \psi_g^C(\vec{x}) \psi^{C'}(\vec{z}_C) \\
&\quad \psi_g^{O[1]}(\vec{y}) \psi^{O[1]'}(\vec{z}_{O1}) \psi_g^{O[2]}(\vec{v}) \psi^{O[2]'}(\vec{z}_{O2}) \psi_g^{S[1]}(\vec{r}) \psi^{S[1]'}(\vec{z}_{S1}) \psi_g^{S[2]}(\vec{s}) \psi^{S[2]'}(\vec{z}_{S2}) \\
&\quad \hat{\xi}_0^\dagger(\vec{x}) \hat{\zeta}_C^\dagger(\vec{z}_C) \hat{\chi}_{[1]0}^\dagger(\vec{y}) \hat{\zeta}_{O1}^\dagger(\vec{z}_{O1}) \hat{\chi}_{[2]0}^\dagger(\vec{v}) \hat{\zeta}_{O2}^\dagger(\vec{z}_{O2}) \\
&\quad \left(\hat{\phi}_{[1]1}^\dagger(\vec{r}) \hat{\phi}_{[2]2}^\dagger(\vec{s}) - \hat{\phi}_{[1]2}^\dagger(\vec{r}) \hat{\phi}_{[2]1}^\dagger(\vec{s}) \right) \hat{\zeta}_{S1}^\dagger(\vec{z}_{S1}) \hat{\zeta}_{S2}^\dagger(\vec{z}_{S2}) |0\rangle.
\end{aligned} \tag{70}$$

Define

$$\begin{aligned}
\widehat{W}_E^S &= \frac{1}{\sqrt{2}} \int d^3\vec{r} d^3\vec{s} d^3\vec{z}_{S1} d^3\vec{z}_{S2} \\
&\quad \left[\psi_g^{S[1]}(\vec{r}) \psi^{S[1]'}(\vec{z}_{S1}) \psi_g^{S[2]}(\vec{s}) \psi^{S[2]'}(\vec{z}_{S2}) \right. \\
&\quad \left(\hat{\phi}_{[1]1}^\dagger(\vec{r}) \hat{\phi}_{[2]2}^\dagger(\vec{s}) - \hat{\phi}_{[1]2}^\dagger(\vec{r}) \hat{\phi}_{[2]1}^\dagger(\vec{s}) \right) \\
&\quad \hat{\zeta}_{S1}^\dagger(\vec{z}_{S1}) \hat{\zeta}_{S2}^\dagger(\vec{z}_{S2}) \\
&\quad - \psi_g^{S[1]*}(\vec{r}) \psi^{S[1]*}(\vec{z}_{S1}) \psi_g^{S[2]*}(\vec{s}) \psi^{S[2]*}(\vec{z}_{S2}) \\
&\quad \hat{\zeta}_{S2}^\dagger(\vec{z}_{S2}) \hat{\zeta}_{S1}^\dagger(\vec{z}_{S1}) \\
&\quad \left. \left(\hat{\phi}_{[2]2}(\vec{s}) \hat{\phi}_{[1]1}(\vec{r}) - \hat{\phi}_{[2]1}(\vec{s}) \hat{\phi}_{[1]2}(\vec{r}) \right) \right]
\end{aligned} \tag{71}$$

and define $\widehat{W}^{O[p]}$, \widehat{W}^C in the same manner as (56) with appropriate changes in variables. If we then take

$$\widehat{V}_E^S = \exp\left(\frac{\pi}{2} \widehat{W}_E^S\right), \quad \widehat{V}^{O[p]} = \exp\left(\frac{\pi}{2} \widehat{W}^{O[p]}\right), \quad \widehat{V}^C = \exp\left(\frac{\pi}{2} \widehat{W}^C\right), \tag{72}$$

and define

$$\widehat{V}^E = \widehat{V}_E^S \widehat{V}^{O[2]} \widehat{V}^{O[1]} \widehat{V}^C, \tag{73}$$

we find that this generates an effectively local Deutsch-Hayden transformation to the vacuum representation,

$$\widehat{V}^{E\dagger} |\psi_{in}^{E'}\rangle = |0\rangle. \tag{74}$$

4 EPRB experiment

From the interpretational rule (25), (26) we see that we will be interested in the expectation value of the density (23) for the $\mathcal{O}[p]$'s states of awareness as well as that of the corresponding operator for \mathcal{C} ,

$$\widehat{\mathcal{N}}_i^C(\vec{x}, t) = \hat{\xi}_i^\dagger(\vec{x}, t) \hat{\xi}_i(\vec{x}, t), \quad i = 0, 1. \tag{75}$$

We will first examine $\widehat{\mathcal{N}}_i^{\mathcal{O}[p]}(\vec{x}, t)$ for t immediately after t_2 , to see if the model gives the correct rates for detection of spin-up systems by the observers $\mathcal{O}[1]$ and $\mathcal{O}[2]$. We will then examine $\widehat{\mathcal{N}}_i^{\mathcal{C}}(\vec{x}, t)$ for $t > t_4$ to calculate the correlation which \mathcal{C} observes between the results obtained by $\mathcal{O}[1]$ and $\mathcal{O}[2]$.

4.1 Measurement of $\mathcal{S}[p]$ by $\mathcal{O}[p]$

Referring to Section 2.2.1 and the interpretational rule (25), (26) we see that we need to calculate

$$\widehat{\chi}_{[p]i}(\vec{x}, t_{[2,3]}) = \widehat{U}_{[0,1]}^\dagger \widehat{U}_{[1,2]}^\dagger \widehat{U}_{[2,3]}^\dagger(t_{[2,3]}) \widehat{\chi}_{[p]i}(\vec{x}) \widehat{U}_{[2,3]}(t_{[2,3]}) \widehat{U}_{[1,2]} \widehat{U}_{[0,1]}. \quad (76)$$

Using (6), (7) and Section (2.2.1), the innermost product in (76) is

$$\widehat{U}_{[2,3]}^\dagger(t_{[2,3]}) \widehat{\chi}_{[p]i}(\vec{x}) \widehat{U}_{[2,3]}(t_{[2,3]}) = \int d^3\vec{y} G^{\mathcal{O}[p]}(\vec{x} - \vec{y}, t_{[2,3]} - t_2) \widehat{\chi}_{[p]i}(\vec{y}), \quad (77)$$

where the Schrödinger Green's function (free-field propagator) for any field is

$$G(\vec{x} - \vec{y}, t - t') = \left(\frac{-2im}{4\pi\hbar(t - t')} \right)^{3/2} \exp \left(\frac{i|\vec{x} - \vec{y}|^2 m}{2\hbar(t - t')} \right) \quad (78)$$

with the mass m appropriate to the field in question. Using (68) and (77),

$$\begin{aligned} & \widehat{U}_{[1,2]}^\dagger \widehat{U}_{[2,3]}^\dagger(t_{[2,3]}) \widehat{\chi}_{[p]i}(\vec{x}) \widehat{U}_{[2,3]}(t_{[2,3]}) \widehat{U}_{[1,2]} = \int d^3\vec{y} G^{\mathcal{O}[p]}(\vec{x} - \vec{y}, t_{[2,3]} - t_2) \\ & \left\{ \widehat{\chi}_{[p]i}(\vec{y}) \left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right)^{2n} \prod_{j=1}^{2n} \int d^3\vec{y}_j f_{[p]}(\vec{y}, \vec{y}_j) \widehat{\mathcal{N}}_{\vec{n}[p],1}^{\mathcal{S}[p]}(\vec{y}_j) \right] \right. \\ & \left. + \widehat{\chi}_{[p]\bar{i}}(\vec{y}) \left[(-1)^{\bar{i}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right)^{2n+1} \prod_{j=1}^{2n+1} \int d^3\vec{y}_j f_{[p]}(\vec{y}, \vec{y}_j) \widehat{\mathcal{N}}_{\vec{n}[p],1}^{\mathcal{S}[p]}(\vec{y}_j) \right] \right\} \quad (79) \end{aligned}$$

where \bar{i} is the complement of i ($\bar{0} = 1, \bar{1} = 0$) Define

$$\begin{aligned} \widehat{Q}_{[p]}(\vec{y}) &= 1 + \sum_{d=1}^{\infty} \frac{(-1)^d}{(2d)!} \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right)^{2d} \prod_{e=1}^{2d} \int d^3\vec{a}_{(e)} d^3\vec{b}_{(e)} d^3\vec{c}_{(e)} \\ & \prod_{f=1}^{2d} f_{[p]}(\vec{y}, \vec{a}_{(f)}) G^{\mathcal{S}[p]*}(\vec{a}_{(f)} - \vec{b}_{(f)}, t_1 - t_0) G^{\mathcal{S}[p]}(\vec{a}_{(f)} - \vec{c}_{(f)}, t_1 - t_0) \prod_{g=1}^{2d} \widehat{\phi}_{\vec{n}[p],[p],1}^\dagger(\vec{b}_{(g)}) \widehat{\phi}_{\vec{n}[p],[p],1}(\vec{c}_{(g)}) \quad (80) \end{aligned}$$

and

$$\begin{aligned} \hat{R}_{[p]}(\vec{y}) &= \sum_{d=0}^{\infty} \frac{(-1)^d}{(2d+1)!} \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right)^{2d+1} \prod_{e=1}^{2d+1} \int d^3 \vec{a}_{(e)} d^3 \vec{b}_{(e)} d^3 \vec{c}_{(e)} \\ &\prod_{f=1}^{2d+1} f_{[p]}(\vec{y}, \vec{a}_{(f)}) G^{S[p]*}(\vec{a}_{(f)} - \vec{b}_{(f)}, t_1 - t_0) G^{S[p]}(\vec{a}_{(f)} - \vec{c}_{(f)}, t_1 - t_0) \prod_{g=1}^{2d+1} \hat{\phi}_{\vec{n}[p],[p],1}^{\dagger}(\vec{b}_{(g)}) \hat{\phi}_{\vec{n}[p],[p],1}(\vec{c}_{(g)}). \end{aligned} \quad (81)$$

Then, using (76) and (79)-(81),

$$\begin{aligned} \hat{\chi}_{[p]i}(\vec{x}, t_{[2,3]}) &= \int d^3 \vec{y} d^3 \vec{z} G^{\mathcal{O}[p]}(\vec{x} - \vec{y}, t_{[2,3]} - t_2) G^{\mathcal{O}[p]}(\vec{y} - \vec{z}, t_1 - t_0) \\ &\left(\hat{\chi}_{[p]i}(\vec{z}) \hat{Q}_{[p]}(\vec{y}) - (-1)^i \hat{\chi}_{[p]\bar{i}}(\vec{z}) \hat{R}_{[p]} \right). \end{aligned} \quad (82)$$

Repeated application of (6)-(8) and use of (27) with (80)-(82) yields

$$\begin{aligned} \hat{\chi}_{[p]0}(\vec{x}, t_{[2,3]}) |\psi_{in}^E\rangle &= \frac{(-1)^{\bar{p}}}{\sqrt{2}} \int d^3 \vec{y} d^3 \vec{z} G^{\mathcal{O}[p]}(\vec{x} - \vec{y}, t_{[2,3]} - t_2) G^{\mathcal{O}[p]}(\vec{y} - \vec{z}, t_1 - t_0) \psi^{\mathcal{O}[p]}(\vec{z}) \\ &\int d^3 \vec{x}_1 d^3 \vec{z}_1 d^3 \vec{v}_1 d^3 \vec{w}_1 \psi_g^{\mathcal{C}}(\vec{x}_1) \psi_g^{\mathcal{O}[\bar{p}]}(\vec{z}_1) \psi_g^{S[p]}(\vec{v}_1) \psi_g^{S[\bar{p}]}(\vec{w}_1) \\ &\left[\left(\hat{\phi}_{[\bar{p}]2}^{\dagger}(\vec{w}_1) \hat{\phi}_{[p]1}^{\dagger}(\vec{v}_1) - \hat{\phi}_{[\bar{p}]1}^{\dagger}(\vec{w}_1) \hat{\phi}_{[p]2}^{\dagger}(\vec{v}_1) \right) + \right. \\ &\left. \left(\cos \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right) - 1 \right) \int d^3 \vec{y}_1 f_{[p]}(\vec{y}, \vec{y}_1) G^{S[p]}(\vec{y}_1 - \vec{v}_1, t_1 - t_0) \right. \\ &\left. \int d^3 \vec{v}_2 G^{S[p]*}(\vec{y}_1 - \vec{v}_2, t_1 - t_0) \hat{\phi}_{\vec{n}[p],[\bar{p}],2}^{\dagger}(\vec{w}_1) \hat{\phi}_{\vec{n}[p],[p],1}^{\dagger}(\vec{v}_2) \right] \hat{\xi}_0^{\dagger}(\vec{x}_1) \hat{\chi}_{[\bar{p}]0}^{\dagger}(\vec{z}_1) |0\rangle \end{aligned} \quad (83)$$

and

$$\begin{aligned} \hat{\chi}_{[p]1}(\vec{x}, t_{[2,3]}) |\psi_{in}^E\rangle &= \frac{(-1)^{\bar{p}}}{\sqrt{2}} \sin \left(\frac{\kappa(t_2 - t_1)}{\hbar} \right) \int d^3 \vec{y} d^3 \vec{z} \\ &G^{\mathcal{O}[p]}(\vec{x} - \vec{y}, t_{[2,3]} - t_2) G^{\mathcal{O}[p]}(\vec{y} - \vec{z}, t_1 - t_0) \psi_g^{\mathcal{O}[p]}(\vec{z}) \\ &\int d^3 \vec{x}_1 d^3 \vec{z}_1 d^3 \vec{v}_1 d^3 \vec{w}_1 \psi_g^{\mathcal{C}}(\vec{x}_1) \psi_g^{\mathcal{O}[\bar{p}]}(\vec{z}_1) \psi_g^{S[p]}(\vec{v}_1) \psi_g^{S[\bar{p}]}(\vec{w}_1) \\ &\int d^3 \vec{y}_1 f_{[p]}(\vec{y}, \vec{y}_1) G^{S[p]}(\vec{y}_1 - \vec{v}_1, t_1 - t_0) \\ &\int d^3 \vec{v}_2 G^{S[p]*}(\vec{y}_1 - \vec{v}_2, t_1 - t_0) \hat{\phi}_{\vec{n}[p],[\bar{p}],2}^{\dagger}(\vec{w}_1) \hat{\phi}_{\vec{n}[p],[p],1}^{\dagger}(\vec{v}_2) \hat{\xi}_0^{\dagger}(\vec{x}_1) \hat{\chi}_{[\bar{p}]0}^{\dagger}(\vec{z}_1) |0\rangle. \end{aligned} \quad (84)$$

where \bar{p} is the complement of p ($\bar{1} = 2, \bar{2} = 1$).

In using (83) and (84) to calculate the integrand in (25), (26) we encounter integrals such as

$$I(\vec{x}) = \int d^3\vec{z} d^3\vec{y} G^{\mathcal{O}[p]*}(\vec{x} - \vec{y}, t_{[2,3]} - t_2) \left(\int d^3\vec{y}_1 f_{[1]}(\vec{y}, \vec{y}_1) |\psi_g^{\mathcal{S}[1]}(\vec{y}_1, t_1)|^2 \right) \psi_g^{\mathcal{O}[1]*}(\vec{y}, t_1), \quad (85)$$

where we have defined, for any system or observer,

$$\psi_g(\vec{x}, t) = \int d^3\vec{y} G(\vec{x} - \vec{y}, t - t_0) \psi_g(\vec{y}). \quad (86)$$

In the MN limit,

$$\lim_{MN} |\psi_g^{\mathcal{S}[1]}(\vec{y}_1, t_1)|^2 = \delta^3(\vec{y}_1 - \vec{x}_{\mathcal{S}[1]}(t_1)). \quad (87)$$

Taking the limit $t_2 = t_1$ and using the Green's-function property

$$\int d^3\vec{y} G(\vec{x} - \vec{y}, t - t') G(\vec{y} - \vec{z}, t' - t'') = G(\vec{x} - \vec{z}, t - t''), \quad (88)$$

we obtain

$$I(\vec{x}) = \psi_g^{\mathcal{O}[1]*}(\vec{x}, t_{[2,3]}) - \tilde{I}(\vec{x}), \quad (89)$$

where

$$\tilde{I}(\vec{x}) = \int_{|\vec{y} - \vec{x}_{\mathcal{S}[1]}(t_1)| > a_{[1]}} d^3\vec{y} G^{\mathcal{O}[1]*}(\vec{x} - \vec{y}, t_{[2,3]} - t_1) \psi_g^{\mathcal{O}[1]*}(\vec{y}, t_1) \quad (90)$$

Using (46) and [62] we obtain the asymptotic equivalence

$$|\tilde{I}(\vec{x})| \sim \pi^{-5/2} \sqrt{2} a_{[1]}^{-3/2} \left(\frac{m_{\mathcal{O}[1]} a_{[1]}^2}{\hbar(t_{[2,3]} - t_2)} \right)^{3/2} \left(\tilde{\alpha}_{\mathcal{O}[1]}(t_1) a_{[1]}^2 \right)^{-1/4} \exp\left(\frac{-\tilde{\alpha}_{\mathcal{O}[1]}(t_1) a_{[1]}^2}{2} \right), \quad \tilde{\alpha}_{\mathcal{O}[1]}(t_1) a_{[1]}^2 \rightarrow \infty, \quad (91)$$

where

$$\tilde{\alpha}_{\mathcal{O}[1]}(t_1) = \alpha_{\mathcal{O}[1]} \left[1 + \frac{\alpha_{\mathcal{O}[1]}^2 \hbar^2 (t_1 - t_0)^2}{m_{\mathcal{O}[1]}^2} \right]^{-1}. \quad (92)$$

So provided $m_{\mathcal{O}[1]}$ does not approach infinity exponentially faster than $\alpha_{\mathcal{O}[1]}$,

$$\lim_{MN} |\tilde{I}(\vec{x})| = 0. \quad (93)$$

We obtain

$$\langle \psi_{in}^E | \widehat{\mathcal{N}}_i^{\mathcal{O}[p]}(\vec{x}, t_{[2,3]}) | \psi_{in}^E \rangle = \frac{1}{2} \delta^3(\vec{x} - \vec{x}_{\mathcal{O}[p]}(t_{[2,3]})), \quad p = 1, 2, i = 0, 1. \quad (94)$$

By the interpretational rule, there is located at $\vec{x}_{\mathcal{O}[p]}(t_{[2,3]})$ an observer who has detected a system spin up along $\vec{n}[p]$, as well as an observer who is still in the ready state. The probability associated with each is 1/2.

4.2 Measurement of $\mathcal{O}[1]$ and $\mathcal{O}[2]$ by \mathcal{C}

In a similar manner, the field operator for the correlation observer \mathcal{C} at $t_{[4,5]}$ is found to be

$$\begin{aligned} \hat{\xi}_i(\vec{x}, t_{[4,5]}) = & \int d^3\vec{y} d^3\vec{y}' d^3\vec{z} d^3\vec{z}' \\ & G^{\mathcal{C}}(\vec{x} - \vec{y}, t_{[4,5]} - t_4) G^{\mathcal{C}}(\vec{y} - \vec{y}', t_3 - t_2) G^{\mathcal{C}}(\vec{y}' - \vec{z}, t_2 - t_1) G^{\mathcal{C}}(\vec{z} - \vec{z}', t_1 - t_0) \\ & \left\{ \hat{\xi}_i(\vec{z}') \left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \left(\frac{\kappa_{\mathcal{C}}(t_4 - t_3)}{\hbar} \right)^{2n} \prod_{j=1}^{2n} \int d^3\vec{y}_j d^3\vec{z}_j f^{\mathcal{C}}(\vec{y}, \vec{y}_j) f^{\mathcal{C}}(\vec{y}, \vec{z}_j) \hat{\Xi}_j \right] \right. \\ & \left. - (-1)^i \hat{\xi}_i(\vec{z}') \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{\kappa_{\mathcal{C}}(t_4 - t_3)}{\hbar} \right)^{2n+1} \prod_{j=1}^{2n+1} \int d^3\vec{y}_j d^3\vec{z}_j f^{\mathcal{C}}(\vec{y}, \vec{y}_j) f^{\mathcal{C}}(\vec{y}, \vec{z}_j) \hat{\Xi}_j \right\}. \end{aligned} \quad (95)$$

where

$$\begin{aligned} \hat{\Xi}_j = & \int d^3\vec{y}_j' G^{\mathcal{O}[1]*}(\vec{y}_j - \vec{y}_j', t_3 - t_2) \int d^3\vec{s}_j' G^{\mathcal{O}[1]*}(\vec{y}_j' - \vec{s}_j', t_1 - t_0) \\ & \left(\hat{\chi}_{[1]1}^\dagger(\vec{s}_j') \hat{Q}_{[1]}(\vec{y}_j') + \hat{\chi}_{[1]0}^\dagger(\vec{s}_j') \hat{R}_{[1]}(\vec{y}_j') \right) \\ & \int d^3\vec{y}_j'' G^{\mathcal{O}[1]}(\vec{y}_j - \vec{y}_j'', t_3 - t_2) \int d^3\vec{s}_j'' G^{\mathcal{O}[1]}(\vec{y}_j'' - \vec{s}_j'', t_1 - t_0) \\ & \left(\hat{\chi}_{[1]1}^\dagger(\vec{s}_j'') \hat{Q}_{[1]}(\vec{y}_j'') + \hat{\chi}_{[1]0}^\dagger(\vec{s}_j'') \hat{R}_{[1]}(\vec{y}_j'') \right) \\ & \int d^3\vec{z}_j' G^{\mathcal{O}[2]*}(\vec{z}_j - \vec{z}_j', t_3 - t_2) \int d^3\vec{p}_j' G^{\mathcal{O}[2]*}(\vec{z}_j' - \vec{p}_j', t_1 - t_0) \\ & \left(\hat{\chi}_{[2]1}^\dagger(\vec{p}_j') \hat{Q}_{[2]}(\vec{z}_j') + \hat{\chi}_{[2]0}^\dagger(\vec{p}_j') \hat{R}_{[2]}(\vec{z}_j') \right) \\ & \int d^3\vec{z}_j'' G^{\mathcal{O}[2]}(\vec{z}_j - \vec{z}_j'', t_3 - t_2) \int d^3\vec{p}_j'' G^{\mathcal{O}[2]}(\vec{z}_j'' - \vec{p}_j'', t_1 - t_0) \\ & \left(\hat{\chi}_{[2]1}^\dagger(\vec{p}_j'') \hat{Q}_{[2]}(\vec{z}_j'') + \hat{\chi}_{[2]0}^\dagger(\vec{p}_j'') \hat{R}_{[2]}(\vec{z}_j'') \right) \end{aligned} \quad (96)$$

To proceed further we resort to perturbation theory. Specifically, we will assume that the coupling which causes \mathcal{C} to change from the ready state if both $\mathcal{O}[1]$ and $\mathcal{O}[2]$ have detected spin-up is weak, so that rather than (45) holding,

$$\lim \kappa_{\mathcal{C}} = \infty, \quad \lim(t_4 - t_3) = 0 \quad \text{s. t.} \quad \lim \left(\frac{\kappa_{\mathcal{C}}(t_4 - t_3)}{\hbar} \right) = \beta, \quad (97)$$

with

$$\beta \ll 1. \quad (98)$$

Focusing on the $i = 1$ case (the case in which \mathcal{C} has determined that both $\mathcal{O}[p]$'s have detected spin-up), we apply the method of Section 4.1 above to (27), (75) and (95)-(98) and obtain, to lowest nonvanishing order in β ,

$$\langle \psi_{in}^E | \widehat{\mathcal{N}}_1^{\mathcal{C}}(\vec{x}, t_{[4,5]}) | \psi_{in}^E \rangle = \beta^2 \frac{1}{4} (1 - \vec{n}[1] \cdot \vec{n}[2]) \delta^3(\vec{x} - \vec{x}_{\mathcal{C}}(t_{[4,5]})). \quad (99)$$

From (99) and the interpretational rule (25), (26) we conclude that at time $t_{[4,5]}$ an observer who has determined both $\mathcal{O}[1]$ and $\mathcal{O}[2]$ to have detected spin-up is located at $\vec{x}_{\mathcal{C}}(t_{[4,5]})$. The probability associated with this observer is $\beta^2 (1/4)(1 - \vec{n}[1] \cdot \vec{n}[2])$. This has the familiar dependence on the relative orientation of the spin analyzers¹¹ which leads to violation of the Clauser-Horne [65] form of Bell's theorem. Of course (99) does *not* violate the Clauser-Horne theorem, due to the perturbative factor $\beta^2 \ll 1$.

5 Summary and discussion

A procedure has been developed for transforming fermionic nonrelativistic Heisenberg-picture quantum field theories to the Deutsch-Hayden picture. The transformation thus obtained is local, at least in the case that the initial spatial dependence can be taken to be well-separated wavepackets, and an explicit form of the transformation is given for initial conditions relevant to the EPRB experiment. A model of effectively local measurement including expressions for interaction Hamiltonians, local representations of observers and an interpretational rule, has been presented and applied to manifestly local calculation of measurements in the EPRB scenario: nonperturbative calculation of spin measurement, and perturbative calculation of spin correlations.

The question posed at the outset—is Everett quantum theory local?—can indeed be answered in the affirmative.

Can the interpretational rule of the present model be modified and extended to apply to more general situations? It should be kept in mind that, in quantum theory, a rule for probability need not apply to all conceivable situations. Page [66]-[68] has argued that probability need only be defined for conscious perceptions. Even if probability can be defined as well for systems involving far less complexity than conscious observers, e.g., records such as scratches on rocks, it may be that all systems for which probability can be defined are macroscopic and localized in a sense related to the idealized limit in the model presented above.

Even if one were to retain some version of the massive narrow-wavepacket limit, it would be of interest to see how to extend this model in other directions, such as to the bosonic and relativistic cases. As presently formulated, the transformation from the Heisenberg to the Deutsch-Hayden picture is only effectively local when the initial state consists of spatially

¹¹See, e.g., [63], [64].

separated wavepackets which are not entangled with each other¹². After the initial time the locality of the model is maintained by the (effective) locality of the equations of motion. But is it possible to implement the Deutsch-Hayden transformation at other times? I suspect this can be done, although at present I do not know how. It would of course be significant if it could be shown that this could *not* be done, thus establishing a connection between locality and initial conditions.

Like the “state reduction” rule of orthodox quantum mechanics, the interpretational rule here is a postulate, although not one that alters the dynamics of the theory. So, the model implicitly defines two levels of “real” entities. The underlying fields from which the model is constructed must be considered as real to satisfy the idea that information is transported by a material carrier from place to place. At the same time, macroscopic objects acquire reality (nonzero probability) by virtue of the configuration of the underlying fields. It would of course be preferable for there to be a single type of reality, with the “higher level” represented here by field bilinears emerging from the lower by means other than postulate. It has been argued, for example, that decoherence favors local densities—which in field theory are represented by field bilinears—due to the conservation laws associated with them [69]–[73].

Wallace and Timpson [74] argue that the Deutsch-Hayden picture does not in fact yield a local version of quantum theory because of its nonuniqueness. (This nonuniqueness is present even in the absence of the fictitious fields employed in the present paper.) They point out that distinct Deutsch-Hayden representations corresponding to the same physical situation correspond to the same set of expectation values, and therefore the same (Heisenberg-picture or Schrödinger-picture) state vector (density operator in the more general case of mixed states). Since the state vector can not in general be decomposed into a direct product of a factor representing what is *here* and factor representing what is *there*, the theory is claimed to be nonlocal.

But this definition of nonlocality is not what is at issue in Bell’s theorem. Rather, it is the possibility of explaining certain expectation values—correlations of distant outcomes—in a locally causal manner. This, as seen above, the Deutsch-Hayden picture is eminently capable of doing.

Acknowledgments

I would like to thank Jianbin Mao and Allen J. Tino for many helpful discussions.

¹² $\mathcal{S}[1]$ and $\mathcal{S}[2]$ are entangled but, at t_0 , have spatially overlapping wavefunctions.

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